Chapter 4

Simultaneous Equations

Learning Objectives

After completing this chapter, you will be able to

- solve simultaneous linear equations in two unknowns by graphical method, method of substitution and method of elimination.
- know the limitation of graphical method.
- formulate and solve simultaneous linear equations in two unknowns.

SY Telecommunication Company provides the following two SMS plans. Which one will you choose? Why?

	Basic monthly charge	Charge for sending each SMS
Plan A	\$20	\$0.4
Plan B	\$0	\$0.5

Preview

[Basic knowledge and techniques required for this chapter.]

A. Basic Knowledge

Relation between ordered pairs and equations

If an ordered pair satisfies a linear equation in two unknowns, the ordered pair is the solution of that equation. For any linear equation in two unknowns, there are infinite number of ordered pairs satisfying it. Thus, all linear equations in two unknowns have infinite number of solutions.

e.g. (1, 2), (3, 0) and $(\frac{5}{2}, \frac{1}{2})$ are solutions of the equation $x+y=3$.

$1+2=3$; $3+0=3$; $\frac{5}{2}+\frac{1}{2}=3$

B. Basic Technique

1. Method of substitution

Example: If $a=3$ and $b=5$, find the value of $5a-b$.

Solution: $5a-b=5(3)-5$
$=\underline{\underline{10}}$

2. Solving linear equations in one unknown

Example: Solve the equation $2x+7=13-x$.

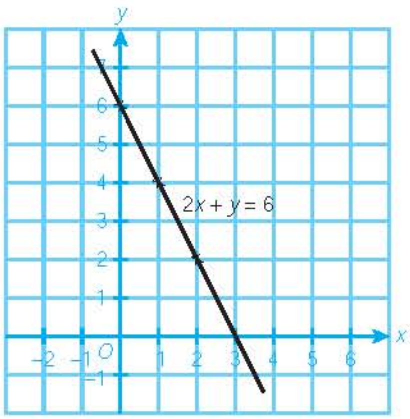
Solution:
$$2x+7=13-x$$
$$2x+x=13-7$$
$$3x=6$$
$$x=\underline{\underline{2}}$$

3. Plotting graphs of linear equations in two unknowns

Example: Plot the graph of the equation $2x+y=6$ on a rectangular coordinate plane.

Solution:

x	y
0	6
1	4
2	2



List some values of x and y which satisfy the equation $2x+y=6$.

According to the information from the table, plot the graph of the equation $2x+y=6$ on a rectangular coordinate plane.

Note: The ordered pair of any point on the graph of an equation must be a solution of the equation. On the other hand, any point represented by an ordered pair satisfying an equation must lie on the graph of the equation.

4.1 Simultaneous Linear Equations in Two Unknowns

Class Activity 4.1

Aim: To learn about simultaneous linear equations in two unknowns and the properties of their solutions

Consider the following problem:

> **Alfred has altogether 19 coins of the *face values* of \$5 and \$10. The total amount is \$140. How many coins of each kind are there?**

Let x and y be the number of \$5 coins and \$10 coins respectively.

1. **(a)** According to the condition 'altogether 19 coins of the face values of \$5 and \$10', set up an equation.

Show Ans.

____

(b) Complete the following table by using the equation obtained in **(a)**.

x	2		10		18
y		11		7	

(c) Are the solutions obtained in **(b)** solutions of the given problem?

☐ Yes ☐ No

2. **(a)** According to the condition 'the total amount is \$140', set up an equation.

Show Ans.

____

(b) Complete the following table by using the equation obtained in **(a)**.

x		8		12	
y	13		9		5

(c) Are the solutions obtained in **(b)** solutions of the given problem?

☐ Yes ☐ No

face value 面額

3. For the solutions obtained in **1(b)** and **2(b)**, which of them is the solution of the given problem? What is its characteristic?

Show Ans.

____

____

Now I see ...

If a question involving two unknowns can be described by two equations, its answer must be an ordered pair satisfying both equations.

A set of two or more equations with same unknowns are called **simultaneous equations**.

Note: In this chapter, 'simultaneous equations' refer to a set of equations with two unknowns and two equations.

In Class Activity 4.1, both x and y must satisfy the equations

$$x + y = 19$$

and $5x + 10y = 140$

We may rewrite them as a set of equations:

$$\begin{cases} x + y = 19 \\ 5x + 10y = 140 \end{cases}$$

◂ '{' means 'simultaneous'.

As all equations in the set are linear equations in two unknowns, we may call them **simultaneous linear equations in two unknowns**.

Extension 4.1

Solution

1. 2 eggs are required to bake a cake and $\frac{1}{2}$ egg is required to bake a tart. A total of 22 eggs are used to bake altogether 20 cakes and tarts.

   Let x and y be the number of cakes and tarts respectively, set up simultaneous linear equations in x and y.

simultaneous equations 聯立方程　　simultaneous linear equations in two unknowns 聯立二元一次方程

Solution

2. Ersenal football club played 38 matches in a *league* last season and got 92 points in total. It is known that each *win* scores 3 points, each *draw* scores 1 point, each *loss* scores 0 point, and Ersenal did not have a loss in the last season.

Let a and b be the number of wins and draws Ersenal had last season respectively, set up simultaneous linear equations in a and b.

4.2 Graphical Method

In Class Activity 4.1, we tried to find the solution of simultaneous equations in two unknowns from a list of ordered pairs. However, this method does not guarantee that one can find a solution. Suppose the solution of the simultaneous linear equations is not on the list of ordered pairs, then this method fails.

In the coming three sections, we will study graphical method and algebraic method. They are more effective in solving simultaneous linear equations.

A Drawing graphs

Class Activity 4.2

Aim: To learn how to use graphical method to solve simultaneous linear equations in two unknowns

1. (a) Plot the graphs of $x+y=8$ and $x-y=2$ on a rectangular coordinate plane.

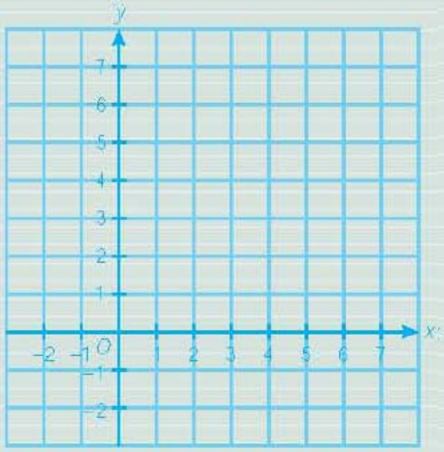
Show Ans.

$x+y=8$

x			
y			

$x-y=2$

x			
y			



(b) Is there any point of intersection on the graph? If yes, write down the coordinates of the point.

____

league 聯賽　　win 勝出　　draw 賽和　　loss 輸

2. Prove that the ordered pair obtained in **1(b)** satisfies the simultaneous equations $\begin{cases} x+y=8 \\ x-y=2 \end{cases}$.

____

3. From the graph, how many solutions do you think are there for the simultaneous equations $\begin{cases} x+y=8 \\ x-y=2 \end{cases}$? Why?

____

Show Ans.

Now I see ...

By plotting the simultaneous linear equations in two unknowns on a rectangular coordinate plane, the point of intersection is the solution of the simultaneous equations.

If the graphs of the simultaneous linear equations in two unknowns intersect at a point, the coordinates of the point satisfy both equations. Thus,

> The coordinates of the point of intersection is the solution of the simultaneous linear equations in two unknowns.

This method is known as **graphical method**. The solution obtained is called the **graphical solution**.

e.g. Consider the simultaneous equations $\begin{cases} x+y=8 \\ x-y=2 \end{cases}$,

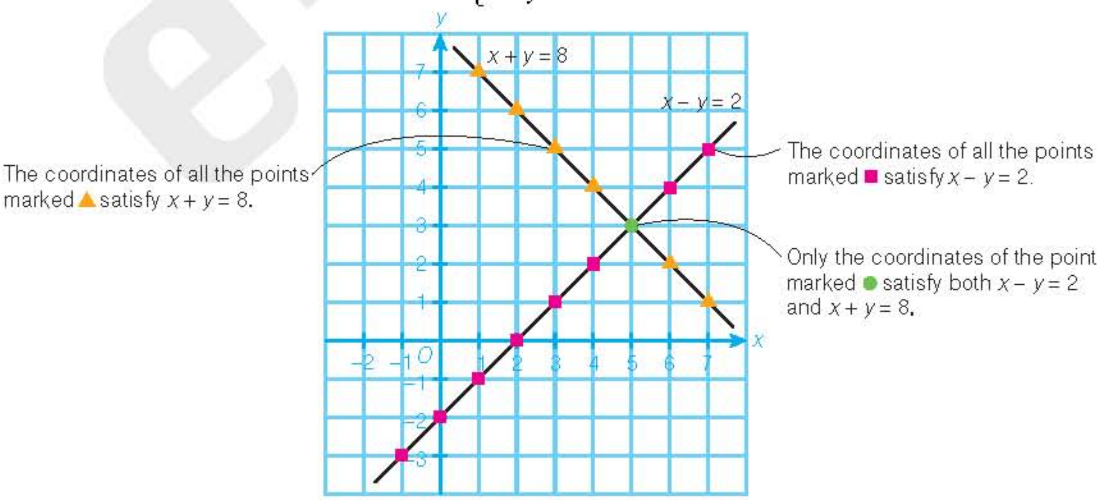


Figure 4.1

From the graph, $x = 5$, $y = 3$ is the solution of the simultaneous equations.

[Refer to page 4.33]

I.T. Tool

graphical method 圖解法　　graphical solution 圖解

Additional Example 4.1

Example 4.1 Using graphical method to obtain integral solution

Solve the simultaneous equations $\begin{cases} 2x + y = 3 \\ x - 2y = 4 \end{cases}$ graphically.

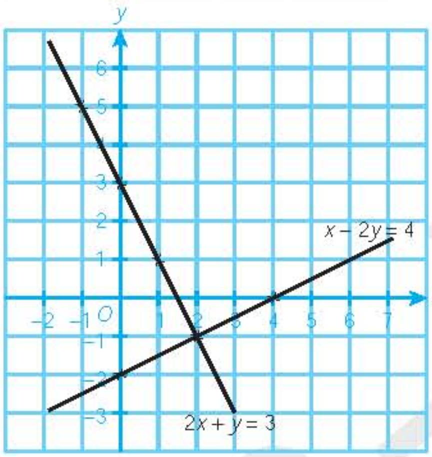
Solution

$2x + y = 3$

x	-1	0	1
y	5	3	1

$x - 2y = 4$

x	0	2	4
y	-2	-1	0



In fact, two points are enough to draw a straight line. However, using three points can ensure the accuracy of the drawing.

In the graph, two straight lines intersect at $(2, -1)$.

$\therefore$ The solution of the simultaneous equations is $x = 2$, $y = -1$.

Notes: (a) Sometimes, the straight lines should be extended to have them intersected.

(b) To ensure the solution obtained is correct, we can substitute the solution into each equation to check whether it satisfies both equations.

Classwork 4.1

Solution

Solve the simultaneous equations $\begin{cases} 3x - y = 2 \\ x + y = 2 \end{cases}$ graphically.

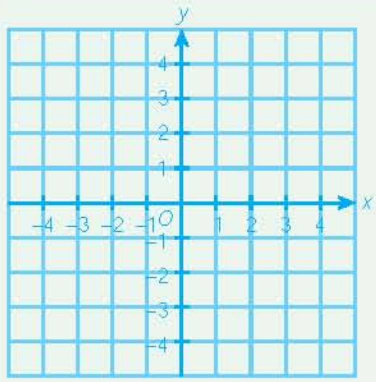
Solution:

$3x - y = 2$

x			
y			

$x + y = 2$

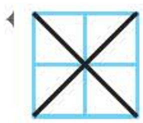
x			
y			

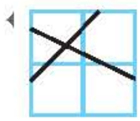


$\therefore$ The solution of the simultaneous equations is ______________.

B Limitation of graphical method

In Example 4.1, the intersection of the simultaneous linear equations in two unknowns just falls on the intersection of the grid lines of the rectangular coordinate plane. Thus, we can easily obtain the solution of the simultaneous equations as $x = 2$, $y = -1$. However, when the intersection of the simultaneous linear equations in two unknowns does not fall on the intersection of the grid lines of the rectangular coordinate plane, we can only estimate the coordinates of the intersection. In most cases, only approximate value, instead of exact value, is obtained by estimation.





e.g. Solve the simultaneous equations $\begin{cases} x - 2y = 0 \\ x + 4y = 10 \end{cases}$ graphically.

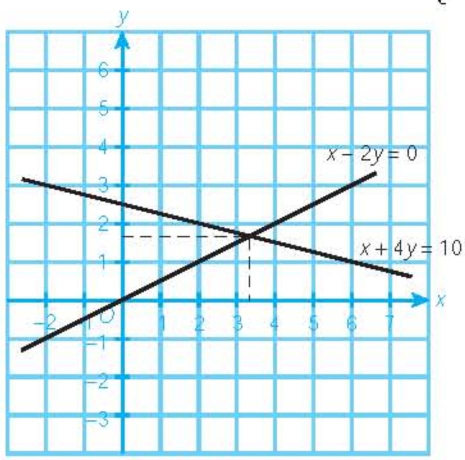


Figure 4.2

In Figure 4.2, it is hard for us to tell the coordinates of the intersection of the graph of the two equations. We can only say that the x-coordinate of the intersection lies between 3 and 4, while the y-coordinate lies between 1 and 2. If a more precise graph paper is used instead, the graphical solution obtained will be more accurate. (See Figure 4.3)

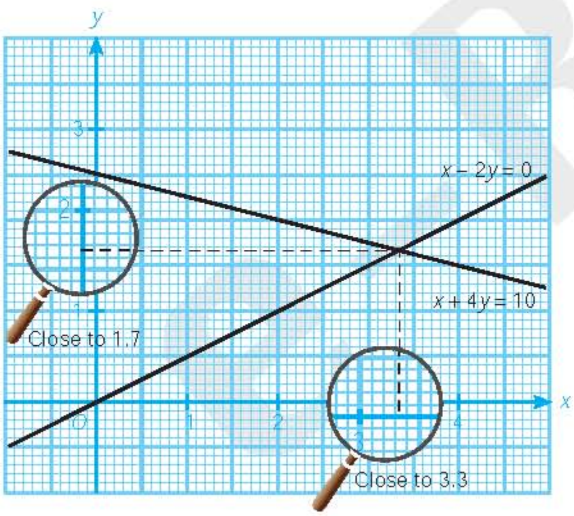


Figure 4.3

From the graph,

x-coordinate of the intersection = 3.3 (corr. to 1 d.p.)

y-coordinate of the intersection = 1.7 (corr. to 1 d.p.)

i.e. (3.3, 1.7) is an approximate solution.

(a) 'corr. to 1 d.p.' is the short form of 'correct to 1 decimal place'.

(b) The exact coordinates of the intersection are $(\frac{10}{3}, \frac{5}{3})$, i.e. $(\frac{10}{3}, \frac{5}{3})$ is an exact solution.

Extension 4.2

Solution

Solve the simultaneous equations $\begin{cases} 2x - y = -1 \\ 5x + 2y = 8 \end{cases}$ graphically. Give your answer correct to 1 decimal place.

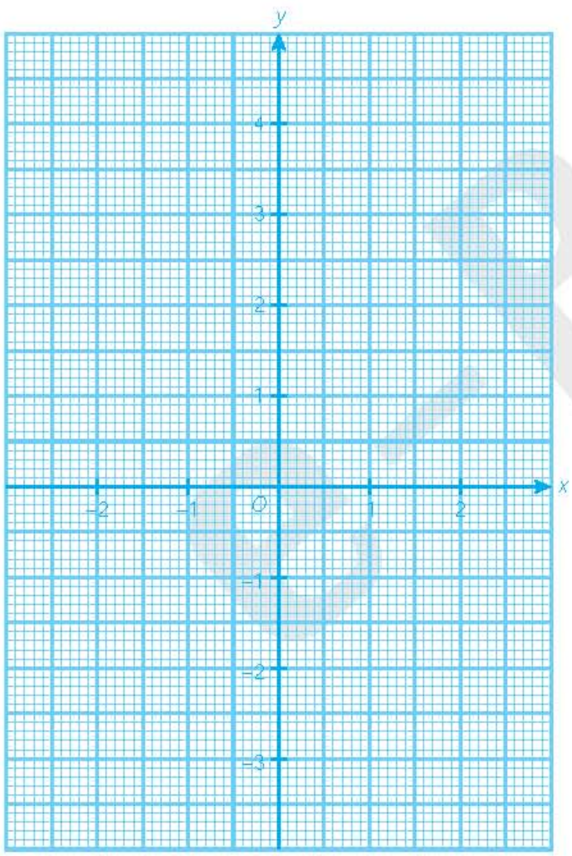
Solution:

$2x - y = -1$

x	-2		1
y		1	

$5x + 2y = 8$

x	0		2
y		0	



$\therefore$ The solution of the simultaneous equations is ________________ (corr. to 1 d.p.).

Skills Upgrading Corner 4.1

Solution

1. Solve the simultaneous equations $\begin{cases} 2x+3y=11 \\ x-y+2=0 \end{cases}$ graphically.

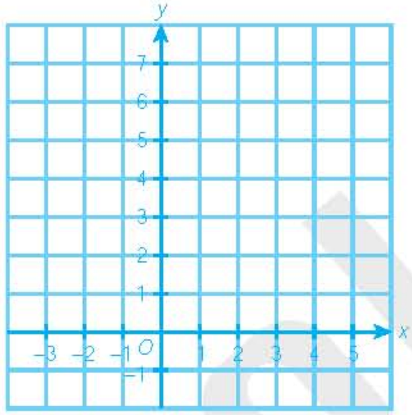
Solution:

$2x+3y=11$

x			
y			

$x-y+2=0$

x			
y			



$\therefore$ The solution of the simultaneous equations is ________________ .

Solution

2. Solve the simultaneous equations $\begin{cases} x+y=3 \\ 5x-2y=6 \end{cases}$ graphically. Give your answer correct to 1 decimal place.

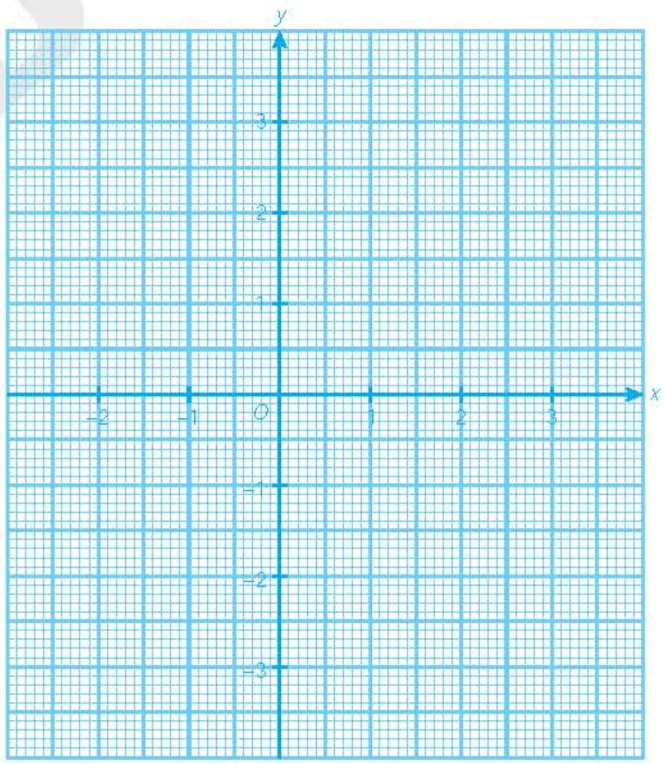
Solution:

$x+y=3$

x		1	
y	3		0

$5x-2y=6$

x	0		2
y		-1	



$\therefore$ The solution of the simultaneous equations is ________________ (corr. to 1 d.p.).

Exercise 4A

Level 1

In each of the following, complete the table and solve the simultaneous equations graphically. (**1** – **3**)

1. $\begin{cases} x+y=0 \\ 3x+4y=4 \end{cases}$

$x + y = 0$

x	−6	−2	0
y			

$3x + 4y = 4$

x	−6	−2	0
y			

2. $\begin{cases} 2x+y=0 \\ 3x+y=-2 \end{cases}$

$2x + y = 0$

x			
y	6	2	0

$3x + y = -2$

x			
y	7	1	−2

3. $\begin{cases} x-y=0 \\ 2x-y=-4 \end{cases}$

$x - y = 0$

x	−5	−3	−1
y			

$2x - y = -4$

x	−5	−3	−2
y			

Solve the following simultaneous equations graphically. (**4** – **6**)

4. $\begin{cases} 2x+y=1 \\ x+2y=5 \end{cases}$

5. $\begin{cases} 4x+y=10 \\ x+y=4 \end{cases}$

6. $\begin{cases} 2x+3y=4 \\ x-y=2 \end{cases}$

Level 2

Solve the following simultaneous equations graphically. Give your answers correct to 1 decimal place. (7 – **9**)

7. $\begin{cases} x+y=6 \\ 5x-3y=9 \end{cases}$

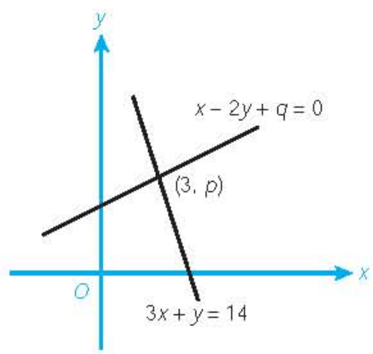
8. $\begin{cases} x+y=4 \\ 4x-3y=6 \end{cases}$

9. $\begin{cases} x+y=5 \\ 4x+y=6 \end{cases}$

10. The figure shows the graph of the simultaneous equations $\begin{cases} 3x+y=14 \\ x-2y+q=0 \end{cases}$. The two straight lines intersect at $(3, p)$.

(a) By considering '$(3, p)$ is a point on the graph of $3x+y=14$', find p.

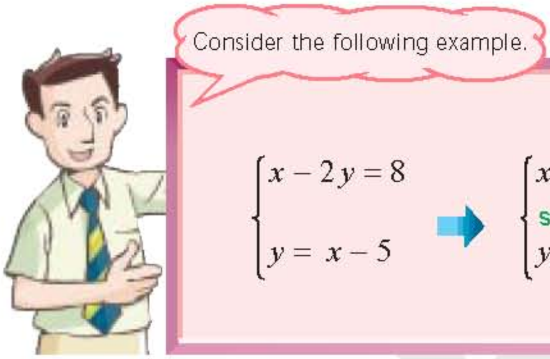
(b) From the result of **(a)**, find q.



4.3 Method of Substitution

In the previous section, we have learned the limitation of graphical method. Now, let us learn some algebraic methods which can give accurate solutions of simultaneous linear equations in two unknowns.

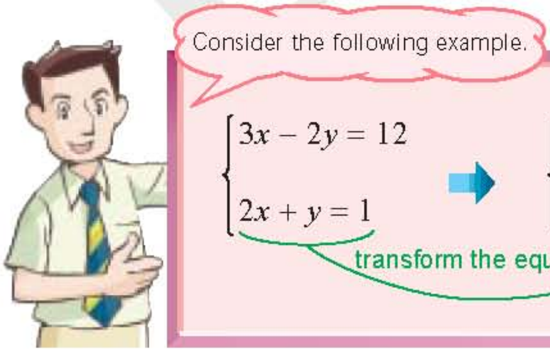
> Under the **method of substitution**, one unknown is eliminated by substituting one equation from the simultaneous linear equations in two unknowns into the other one.



$$\begin{cases} x-2y=8 \\ y=x-5 \end{cases} \Rightarrow \begin{cases} x-2y=8 \\ y=x-5 \end{cases} \xrightarrow{\text{eliminate } y} x-2(x-5)=8 \xrightarrow{\text{solve the equation}} \cdots$$

substitute

In the calculation, we may have to transform one equation, by putting one unknown on the left-hand side of the equation, before applying the method of substitution.



$$\begin{cases} 3x-2y=12 \\ 2x+y=1 \end{cases} \Rightarrow \begin{cases} 3x-2y=12 \\ y=1-2x \end{cases} \xrightarrow{\text{eliminate } y} 3x-2(1-2x)=12 \xrightarrow{\text{solve the equation}} \cdots$$

substitute

transform the equation

method of substitution 代入消元法

Additional Example 4.2

Example 4.2 Transforming once before substitution

Use the method of substitution to solve the simultaneous equations $\begin{cases}2x+3y=5\\x-y=5\end{cases}$.

Solution

[Analysis: Since the equation $x-y=5$ is simpler, we put x on the left-hand side of the equation before applying the method of substitution.]

$$\begin{cases}2x+3y=5 \text{ .... (1)}\\x-y=5 \text{ .... (2)}\end{cases}$$

From (2), $x=y+5$ .... (3)

Substitute (3) into (1),

$$\begin{aligned}2(y+5)+3y&=5\\2y+10+3y&=5\\5y&=-5\\y&=-1\end{aligned}$$

Substitute $y=-1$ into (3),

$$\begin{aligned}x&=-1+5\\&=4\end{aligned}$$

$\therefore$ The solution of the simultaneous equations is $x=4$, $y=-1$.

Check the answer:
When $x=4$ and $y=-1$,
for equation (1),
L.H.S. $=2(4)+3(-1)=5=$ R.H.S.
for equation (2),
L.H.S. $=4-(-1)=5=$ R.H.S.
$\therefore$ $x=4$, $y=-1$ is a correct solution.

Classwork 4.2

Solution

Use the method of substitution to solve the following simultaneous equations.

(a) $\begin{cases}3x+y=13\\x-y=3\end{cases}$

(b) $\begin{cases}5x+7y=18\\x+y=6\end{cases}$

Additional Example 4.3

Example 4.3 Transforming before substituting more complicated expression

Use the method of substitution to solve the simultaneous equations $\begin{cases}2x-3y=3\\3x+4y=13\end{cases}$.

Solution

[Analysis: Since the equation $2x-3y=3$ is simpler, we put x on the left-hand side of the equation before applying the method of substitution.]

$$\begin{cases}2x-3y=3 \text{ .... (1)}\\3x+4y=13 \text{ .... (2)}\end{cases}$$

From (1), $2x=3y+3$

$$x=\frac{3y+3}{2} \text{ .... (3)}$$

Classwork 4.3

Solution

Use the method of substitution to solve the following simultaneous equations.

(a) $\begin{cases}2x=7y-12\\2y+3x=7\end{cases}$

(b) $\begin{cases}3x-2y=7\\4x-3y=-2\end{cases}$

Substitute (3) into (2),

$$3\left(\frac{3y+3}{2}\right)+4y=13$$

$$2\left[3\left(\frac{3y+3}{2}\right)\right]+2(4y)=2(13)$$

◂ Multiply both sides by 2 can eliminate the denominator.

$$9y+9+8y=26$$
$$17y=17$$
$$y=1$$

Substitute $y=1$ into (3),

$$x=\frac{3(1)+3}{2}$$
$$=3$$

$\therefore$ The solution of the simultaneous equations is $x=3$, $y=1$.

◂ Check the answer.

Additional Example 4.4

Example 4.4 Setting up simultaneous equations from an equality

Use the method of substitution to solve the simultaneous equations $4x+y-9=2x-y-3=0$.

Classwork 4.4

Solution

Use the method of substitution to solve the simultaneous equations $-x+y+17=x+2y=16$.

Solution

[Analysis: $a=b=c$ ➡ $a=c$ and $b=c$ ➡ $\begin{cases} a=c \\ b=c \end{cases}$]

Rewrite the simultaneous equations as follows:

$$\begin{cases} 4x+y-9=0 \text{..........................(1)} \\ 2x-y-3=0 \text{..........................(2)} \end{cases}$$

From (2), $y=2x-3$..................(3)

Substitute (3) into (1),

$$4x+(2x-3)-9=0$$
$$4x+2x-3-9=0$$
$$6x-12=0$$
$$6x=12$$
$$x=2$$

Substitute $x=2$ into (3),

$$y=2(2)-3$$
$$=1$$

$\therefore$ The solution of the simultaneous equations is $x=2$, $y=1$.

◂ Check the answer.

Skills Upgrading Corner 4.2

Solution

Use the method of substitution to solve the following simultaneous equations.

(a) $6x+5y-4=3y-8x-14=0$

(b) $\begin{cases}\dfrac{x}{2}-y=5\\4x-3y=10\end{cases}$

Exercise 4B

Level 1

Use the method of substitution to solve the following simultaneous equations. (1 – 5)

1. (a) $\begin{cases}2x+y=9\\y=x-3\end{cases}$ (b) $\begin{cases}x=4y+2\\x+2y=8\end{cases}$ (c) $\begin{cases}x+2y=9\\y=2x-3\end{cases}$

2. (a) $\begin{cases}x-2y=1\\x+y=7\end{cases}$ (b) $\begin{cases}x-y=1\\x+2y=4\end{cases}$ (c) $\begin{cases}x+y=8\\3x+y=18\end{cases}$

3. (a) $\begin{cases}x+2y=12\\x-5y=-2\end{cases}$ (b) $\begin{cases}3x+y=-1\\9x+2y=1\end{cases}$ (c) $\begin{cases}x-3y=-2\\5x-2y=16\end{cases}$

4. (a) $\begin{cases}2x-3y=1\\4x-5y=5\end{cases}$ (b) $\begin{cases}7x-2y=2\\3x-4y=-7\end{cases}$ (c) $\begin{cases}5x+6y=-2\\2x-3y=10\end{cases}$

5. (a) $\begin{cases}3x+4y+2=0\\6x+5y+7=0\end{cases}$ (b) $\begin{cases}5x-2y+19=0\\3x+4y+1=0\end{cases}$ (c) $\begin{cases}2x+5y-8=0\\9x+10y-11=0\end{cases}$

Level 2

Use the method of substitution to solve the following simultaneous equations. (6 – 9)

6. (a) $\begin{cases}5x+3y+6=0\\3x-2y-23=0\end{cases}$ (b) $\begin{cases}3x+5y-1=0\\4x+7y-5=0\end{cases}$ (c) $\begin{cases}3x-4y-23=0\\2x+3y-4=0\end{cases}$

7. (a) $3x+2y=x-6=0$ (b) $7x+2y-7=3x-y=15$ (c) $5x-3y-11=4x-7y-18=0$

8. (a) $\begin{cases}2(x+y)-3x=4\\3(x-y)+5y=12\end{cases}$ (b) $\begin{cases}2x-3y=5\\15x-(2y-3)=20\end{cases}$ (c) $\begin{cases}4(x-1)-5(2-y)=0\\26x+3y+27=0\end{cases}$

9. (a) $\begin{cases}3x-2y=2\\x-\dfrac{y}{2}=-4\end{cases}$ (b) $\begin{cases}\dfrac{3x}{2}+y=15\\9x-2y=18\end{cases}$ (c) $\begin{cases}\dfrac{x}{3}-2y=3\\2x+y=5\end{cases}$

4.4 Method of Elimination

Using the method of substitution in solving simultaneous linear equations in two unknowns may involve complicated calculations with fractions. In this case, the method of elimination may make calculations easier.

> Under the **method of elimination**, one unknown is eliminated by adding/subtracting one equation from the simultaneous linear equations in two unknowns to/from the other one.

Consider the following example.

$$\begin{cases} 2x + 3y = 1 \\ 5x - 3y = 34 \end{cases} \Rightarrow \begin{array}{rrrl} & 2x & +3y & = 1 \\ +) & 5x & -3y & = 34 \\ \hline & 7x & & = 35 \end{array}$$

y is eliminated after addition

In the calculation, we may have to multiply the equation(s) before applying the method of elimination.

Consider the following example.

$$\begin{cases} 3x + 2y = 11 \\ x + y = 4 \end{cases} \xrightarrow{\times 2} \begin{cases} 3x + 2y = 11 \\ 2x + 2y = 8 \end{cases} \Rightarrow \begin{array}{rrrl} & 3x & +2y & = 11 \\ -) & 2x & +2y & = 8 \\ \hline & x & & = 3 \end{array}$$

y is eliminated after subtraction

method of elimination 加減消元法

Example 4.5 Direct addition/subtraction

Additional Example 4.5

Use the method of elimination to solve the simultaneous equations
$$\begin{cases} 5x+2y=8 \\ 5x-4y=14 \end{cases}.$$

Solution

[Analysis: Since both equations have a common term $5x$, x can be eliminated by subtraction.]

$$\begin{cases} 5x+2y=8 \text{ .... } (1) \\ 5x-4y=14 \text{ .... } (2) \end{cases}$$

(1) − (2),

$$\begin{aligned} (5x+2y)-(5x-4y) &= 8-14 \\ 5x+2y-5x+4y &= -6 \\ 6y &= -6 \\ y &= -1 \end{aligned}$$

◂ Eliminate x.

Substitute $y=-1$ into (1),

$$\begin{aligned} 5x+2(-1) &= 8 \\ 5x &= 10 \\ x &= 2 \end{aligned}$$

∴ The solution of the simultaneous equations is $x=2$, $y=-1$.

◂ Check the answer.

Classwork 4.5

Solution

Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases} 7x+6y=19 \\ 7x+y=9 \end{cases}$

(b) $\begin{cases} 4x-3y=3 \\ x+3y=12 \end{cases}$

Example 4.6 Multiplying an equation before addition/subtraction

Additional Example 4.6

Use the method of elimination to solve the simultaneous equations
$$\begin{cases} 3x-y=11 \\ 5x+3y=9 \end{cases}.$$

Solution

[Analysis: Obtain a common term $3y$ for both equations by multiplying both sides of $3x-y=11$ by 3.]

$$\begin{cases} 3x-y=11 \text{ .... } (1) \\ 5x+3y=9 \text{ .... } (2) \end{cases}$$

(1) × 3,

$$\begin{aligned} (3x-y)\times 3 &= 11\times 3 \\ 9x-3y &= 33 \text{ .... } (3) \end{aligned}$$

Classwork 4.6

Solution

Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases} 2x+y=7 \\ x-2y=-4 \end{cases}$

(b) $\begin{cases} 4x+y=5 \\ 3x+4y=7 \end{cases}$

(3) + (2),

$$(9x-3y)+(5x+3y)=33+9$$
$$9x-3y+5x+3y=42$$
$$14x=42$$
$$x=3$$

Substitute $x=3$ into (1),

$$3(3)-y=11$$
$$-y=2$$
$$y=-2$$

$\therefore$ The solution of the simultaneous equations is $x=3,\ y=-2$.

◂ Check the answer.

Additional Example 4.7

Example 4.7 Multiplying both equations before addition/subtraction

Use the method of elimination to solve the simultaneous equations $\begin{cases}3x+4y=3\\5x-7y=46\end{cases}$.

Solution

[Analysis: Obtain a common term for both equations by multiplying each equation by an appropriate number. For example, multiply both sides of $3x+4y=3$ by 5 and multiply both sides of $5x-7y=46$ by 3 to obtain a common term $15x$ for the two equations.]

$$\begin{cases}3x+4y=3\ \ldots\ldots\ldots\ldots(1)\\5x-7y=46\ \ldots\ldots\ldots\ldots(2)\end{cases}$$

(1) × 5,

$$15x+20y=15\ \ldots\ldots\ldots\ldots(3)$$

(2) × 3,

$$15x-21y=138\ \ldots\ldots\ldots\ldots(4)$$

(3) − (4),

$$(15x+20y)-(15x-21y)=15-138$$
$$15x+20y-15x+21y=-123$$
$$41y=-123$$
$$y=-3$$

Substitute $y=-3$ into (1),

$$3x+4(-3)=3$$
$$3x=15$$
$$x=5$$

$\therefore$ The solution of the simultaneous equations is $x=5,\ y=-3$.

◂ Check the answer.

Classwork 4.7

Solution

Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases}2x+3y=8\\3x+2y=7\end{cases}$

(b) $\begin{cases}5x+3y=4\\7x+2y=10\end{cases}$

Alternative method: (eliminate y)

$(1)\times 7$,

$21x+28y=21$ .... (3)

$(2)\times 4$,

$20x-28y=184$ .... (4)

$(3)+(4)$,

$$(21x+28y)+(20x-28y)=21+184$$
$$21x+28y+20x-28y=205$$
$$41x=205$$
$$x=5$$

Substitute $x=5$ into (1),

$$3(5)+4y=3$$
$$4y=-12$$
$$y=-3$$

$\therefore$ The solution of the simultaneous equations is $x=5,\ y=-3$.

Check the answer.

Additional Example 4.8

Example 4.8 Equations involving fractions

Use the method of elimination to solve the simultaneous equations $\frac{x+3}{5}-\frac{8-y}{4}=3x+5y-16=0$.

Solution

[Analysis: Since fractions are involved in the equation $\frac{x+3}{5}-\frac{8-y}{4}=0$, we can simply eliminate the denominators in the equation by multiplying both sides of the equation by the L.C.M. of all denominators, i.e. multiplied by 20 which is the L.C.M. of 5 and 4.]

Rewrite the simultaneous equations as follows:

$$\begin{cases}\frac{x+3}{5}-\frac{8-y}{4}=0 \ldots\ldots\ldots\ldots\ldots (1)\\ 3x+5y-16=0 \ldots\ldots\ldots\ldots\ldots (2)\end{cases}$$

$(1)\times 20$,

$$\frac{x+3}{5}\times 20-\frac{8-y}{4}\times 20=0\times 20$$
$$4(x+3)-5(8-y)=0$$
$$4x+12-40+5y=0$$
$$4x+5y-28=0 \ldots\ldots (3)$$

Classwork 4.8

Solution

Use the method of elimination to solve the simultaneous equations $\frac{x-2}{2}+\frac{y-2}{7}=x+5y+21=0$.

(3) – (2),

$$\begin{aligned}(4x+5y-28)-(3x+5y-16)&=0\\ 4x+5y-28-3x-5y+16&=0\\ x-12&=0\\ x&=12\end{aligned}$$

Substitute $x=12$ into (2),

$$\begin{aligned}3(12)+5y-16&=0\\ 5y&=-20\\ y&=-4\end{aligned}$$

$\therefore$ The solution of the simultaneous equations is $x=12,\ y=-4$.

◂ Check the answer.

Skills Upgrading Corner 4.3

Solution

Use the method of elimination to solve the simultaneous equations
$\dfrac{2x+5y}{3}-\dfrac{x+y+1}{5}=\dfrac{x+5}{2}-\dfrac{y-4}{5}-5=-4$.

Exercise 4C

Level 1

Use the method of elimination to solve the following simultaneous equations. (**1** – **5**)

1. **(a)** $\begin{cases}x+y=4\\x-y=2\end{cases}$ **(b)** $\begin{cases}2x+y=7\\3x-y=8\end{cases}$ **(c)** $\begin{cases}3x+y=11\\-3x+2y=10\end{cases}$

2. **(a)** $\begin{cases}2x+y=5\\x+y=3\end{cases}$ **(b)** $\begin{cases}2x+5y=7\\2x+y=5\end{cases}$ **(c)** $\begin{cases}5x-y=16\\5x-3y=8\end{cases}$

3. **(a)** $\begin{cases}x-y=1\\2x+3y=7\end{cases}$ **(b)** $\begin{cases}x+2y=6\\2x+y=9\end{cases}$ **(c)** $\begin{cases}5x+3y=10\\x+2y=9\end{cases}$

4. **(a)** $\begin{cases}4x+7y=3\\2x+5y=3\end{cases}$ **(b)** $\begin{cases}6x-5y=26\\3x-2y=14\end{cases}$ **(c)** $\begin{cases}10x-3y=69\\2x-9y=39\end{cases}$

5. **(a)** $\begin{cases}5x+2y=28\\3x+5y=51\end{cases}$ **(b)** $\begin{cases}5x-3y=30\\7x+4y=1\end{cases}$ **(c)** $\begin{cases}15x+7y=-5\\11x+10y=-28\end{cases}$

Level 2

Use the method of elimination to solve the following simultaneous equations. (**6** – **8**)

6. **(a)** $\begin{cases} 5x - y = 13 \\ y - 3x = -9 \end{cases}$ **(b)** $\begin{cases} 4x - 3y = 7 \\ y - 4x = -13 \end{cases}$ **(c)** $\begin{cases} 3x - y = -12 \\ 3y - 2x = 22 \end{cases}$

7. **(a)** $\begin{cases} 2.5x - 1.5y = 9 \\ 0.5x + 2.5y = 13 \end{cases}$ **(b)** $\begin{cases} \dfrac{x}{2} + \dfrac{3y}{2} = 1 \\ x + y = 4 \end{cases}$ **(c)** $\begin{cases} 3x + 5y = 75 \\ \dfrac{x}{2} - y = -4 \end{cases}$

8. **(a)** $3x + 7y = x - 3y = 8$ **(b)** $\dfrac{2x + 3y}{6} = \dfrac{6x - y}{3} = 4$ **(c)** $\dfrac{x}{3} - \dfrac{y}{4} = 2x + y = 4$

9. **(a)** Use the method of elimination to solve the simultaneous equations $\begin{cases} 2x - 5y = 5 \\ 3x + 5y = 20 \end{cases}$.

(b) Let $x = 0.1a$ and $y = 0.01b$. By the result of **(a)**, solve the simultaneous equations $\begin{cases} 0.2a - 0.05b = 5 \\ 0.3a + 0.05b = 20 \end{cases}$.

10. **(a)** Use the method of elimination to solve the simultaneous equations $\begin{cases} x + 2y = -2 \\ x - 3y = 8 \end{cases}$.

(b) Let $x = \dfrac{1}{a}$ and $y = \dfrac{1}{b}$. By the result of **(a)**, solve the simultaneous equations $\begin{cases} \dfrac{1}{a} + \dfrac{2}{b} = -2 \\ \dfrac{1}{a} - \dfrac{3}{b} = 8 \end{cases}$.

4.5 Applications of Simultaneous Linear Equations in Two Unknowns

Daily-life problems involving two unknowns can be solved by using simultaneous linear equations in two unknowns.

As early as about AD100, problems involving simultaneous linear equations in two unknowns were already recorded in a chapter on equations of a Chinese Mathematics Classics '*Jiu Zhang Suan Shu*'.

Jiu Zhang Suan Shu 九章算術

Additional Example 4.9

Example 4.9 Forming equations by adding/subtracting unknowns directly

The brother and sister have altogether 48 stamps. If the sister has 16 stamps more than the brother's, how many stamps does each of them have?

Solution

Let x and y be the number of stamps the sister and brother have respectively.

Accordingly,

$$\begin{cases} x+y=48 \text{ ....}(1) \\ x-y=16 \text{ ....}(2) \end{cases}$$

(1) + (2),

$$(x+y)+(x-y)=48+16$$
$$2x=64$$
$$x=32$$

Substitute $x=32$ into (1),

$$32+y=48$$
$$y=16$$

$\therefore$ The sister has 32 stamps and the brother has 16 stamps.

Steps to solve a problem:
1. Let x and y be the unknowns.
2. Set up equations.
3. Solve equations.
4. Check the answer.
   The sister has 32 stamps. The brother has 16 stamps. They have altogether $32+16=48$ stamps. The sister has $32-16=16$ more stamps than the brother's.
5. Answer the question.

Classwork 4.9

Solution

Mimi has 32 more game cards than her brother's. If the quantity of her brother's game cards triples, they have altogether 44 game cards. How many game cards does each of them have originally?

Additional Example 4.10

Example 4.10 Forming equations by multiplying unknowns before addition/subtraction

3 tables and 4 chairs are sold at \$6 400, while 4 tables and 3 chairs are sold at \$6 900. What are the respective selling prices of a table and a chair?

Solution

Let $\$x$ be the selling price of a table and $\$y$ be the selling price of a chair.

Accordingly,

$$\begin{cases} 3x+4y=6\ 400 \text{ ....}(1) \\ 4x+3y=6\ 900 \text{ ....}(2) \end{cases}$$

(1) $\times$ 3,

$$9x+12y=19\ 200 \text{ ....}(3)$$

(2) $\times$ 4,

$$16x+12y=27\ 600 \text{ ....}(4)$$

Classwork 4.10

Solution

4 apples and 3 oranges are sold at \$17, while 6 apples and 5 oranges are sold at \$27. What are the respective selling prices of an apple and an orange?

(4) – (3),

$(16x+12y)-(9x+12y)=27\ 600-19\ 200$

$16x+12y-9x-12y=8\ 400$

$7x=8\ 400$

$x=1\ 200$

Substitute $x=1\ 200$ into (1),

$3(1\ 200)+4y=6\ 400$

$4y=2\ 800$

$y=700$

∴ The selling price of a table is \$1 200, and the selling price of a chair is \$700.

◂ Check the answer.

Additional Example 4.11

Example 4.11 Using the concept of ratio

The original number of candies with Maggie and Charles was in the ratio of 9 : 7. If Maggie gave 12 candies to Charles, the ratio became 3 : 5. How many candies did each of them get originally?

Classwork 4.11

Solution

The ages of two brothers are in the ratio of 3 : 2. After 8 years, the ratio will become 5 : 4. What is the current age of each of them?

Solution

Let x and y be the respective number of candies Maggie and Charles have got originally.

Accordingly,

$$\begin{cases} \dfrac{x}{y}=\dfrac{9}{7} \quad \cdots\cdots (1) \\ \dfrac{x-12}{y+12}=\dfrac{3}{5} \quad \cdots\cdots (2) \end{cases}$$

$\because\ a : b = \dfrac{a}{b}$

$\therefore\ 9 : 7 = \dfrac{9}{7},\ 3 : 5 = \dfrac{3}{5}$

From (1), $7x=9y$ ……………………… (3)

From (2), $5x-60=3y+36$

$5x=3y+96$ ………… (4)

(4) × 3, $15x=9y+288$ ………………… (5)

(5) – (3), $15x-7x=(9y+288)-9y$

$8x=288$

$x=36$

Substitute $x=36$ into (3),

$7(36)=9y$

$y=28$

∴ Originally, Maggie has got 36 candies and Charles has got 28 candies.

◂ Check the answer.

Skills Upgrading Corner 4.4

Solution

1. Kenneth paid $36 for cocktail buns and pineapple buns. The cocktail buns cost $2 each and pineapple buns cost $2.5 each. If the number of cocktail buns were 9 more than that of pineapple buns, how many buns of each kind did Kenneth buy?

Solution

2. The sum of the units digit and the tens digit of a two-digit number is 16. If the two digits are *interchanged*, the new number is 18 less than the original number. Find the original number.

Exercise 4D

Level 1

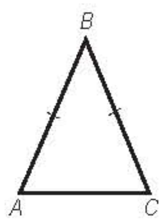
1. If the sum of two numbers is 74 and their difference is 24, find the two numbers.

2. The age of Alex is 3 times of his daughter's. The difference between their ages is 36. Find the respective ages of Alex and his daughter.

3. The difference between the ages of Christina and her son is 30. After 3 years, her age will be 3 times of her son's. Find the current age of Christina.

4. The perimeter of a rectangle is 48 cm. If the length is 5 times of the width, find the length and width of the rectangle.

5. The figure shows an isosceles triangle ABC with $AB = BC$. If the perimeter of ΔABC is 96 cm, and AB is 12 cm longer than AC, find the length of AC.



6. There are 200 seats available in Creative Theatre. An adult ticket is sold for $50, while a student ticket is sold for $20. If the theatre has a *full house* for a performance with a total income of $5 770, how many adult audience are there for this performance?

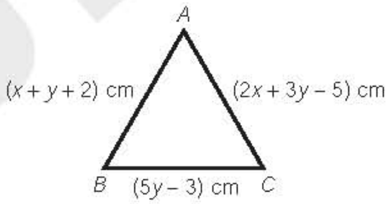
interchange 交換 full house 滿座

7. Paul has some 50¢ coins and \$2 coins. The total value of the coins is \$26. If there are 22 coins altogether, how many coins of each kind does he have?

8. Michael pays \$46 for 2 lunch boxes and 1 bottle of orange juice. Wendy pays \$74 for 3 lunch boxes and 2 bottles of orange juice. How much do the lunch boxes and bottles of orange juice cost each?

9. 5 ice-creams and 3 soft drinks are sold at \$84. 3 ice-creams and 7 soft drinks are sold at \$92. How much do the ice-creams and soft drinks cost each?

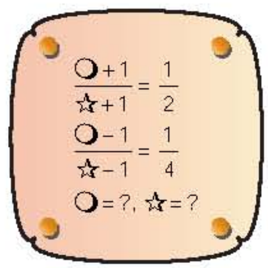
10. The figure shows an equilateral triangle ABC.
   (a) Find x and y.
   (b) Find the perimeter of ΔABC.



Level 2

11. Mrs. Lee has bought 3 shirts and 5 pairs of pants in a boutique for \$1 110. If she gets 2 more shirts and takes 2 pairs of pants out, she will pay \$60 less. What are the respective selling prices of a shirt and a pair of pants?

12. According to the equations in the figure, find the values of ❍ and ☆.



13. The sum of the units digit and the tens digit of a two-digit number is 13. If the two digits are interchanged, the new number is 45 greater than the original number. Find the original number.

14. The sum of the units digit and the tens digit of a two-digit number is 7. If 3 is added to the number, the new number equals 11 times the tens digit of the original number. Find the original number.

15. The sum of the hundreds digit and the units digit of a three-digit number is 12. If the hundreds digit and the units digit are interchanged, the new number is 594 less than the original number. It is given that the tens digit of the original number is 5, find the original number.

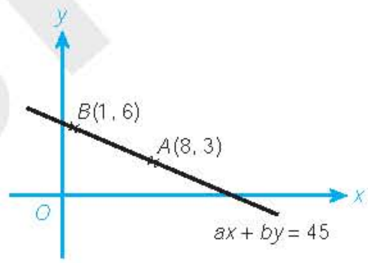
16. A sum of money is distributed to Cherrie and Terry in the ratio of $7:4$. If Cherrie gives \$600 to Terry, the ratio will become $5:6$.

(a) Find the original amount distributed to each of them.

(b) Find the total amount of the sum of money.

17. The ages of Mr. Chan and Wesley are in the ratio of $7:3$. After 5 years, the ratio will become 2 : 1. What are their current ages each?

18. The figure shows the graph of the linear equation in two unknowns $ax + by = 45$. It is given that $A(8, 3)$ and $B(1, 6)$ are two points on the graph, find a and b.



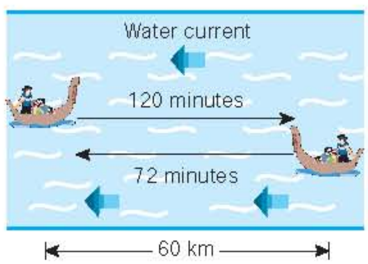
19. The prices of Grade A rice and Grade B rice are \$7/kg and \$2/kg respectively. A total of 30 kg of the two types of rice are mixed together, and the price of the mixed rice is \$4/kg. What is the weight of each type of rice in the mixed rice?

20. The selling price of 6 kg of Tea A and 10 kg of Tea B is \$600. It is known that the selling price of 35 kg of Tea A is the same as that of 25 kg of Tea B.

(a) What is the selling price of each kind of tea per kg?

(b) What is the selling price of 6 kg of Tea A?

21. A boat travels 60 km *upstream* in 120 minutes, and returns to the starting spot *downstream* in 72 minutes. It is given that the flow speed of the water current is x km/h, and the speed of the boat in still water is y km/h.



(a) Express, in terms of x and y, the speed of the boat travelling upstream.

(b) Express, in terms of x and y, the speed of the boat travelling downstream.

(c) Find the flow speed of the water current and the speed of the boat in still water.

upstream 逆流　　downstream 順流

Chapter Summary

Chapter Summary

Fact to Remember

Methods for solving simultaneous linear equations in two unknowns:

1. Graphical method
2. Method of substitution
3. Method of elimination

Check Yourself

[This is a quiz to remind you of the basic concepts you have learned in this chapter. Each question tests a concept under the section listed on the right. Failure in any part of a question indicates a need to do a revision on the section listed.]

Section

1. Solve the following simultaneous equations graphically. 4.2A

   (a) $\begin{cases} 6x - y = 13 \\ x + y = 1 \end{cases}$ **(b)** $\begin{cases} 2x + 3y = 13 \\ 3x + 2y = 17 \end{cases}$

2. Use the method of substitution to solve the following simultaneous equations. 4.3

   (a) $\begin{cases} x + y = 10 \\ x = 3y - 2 \end{cases}$ **(b)** $\begin{cases} 3x + 2y = -1 \\ 2x + 3y = 6 \end{cases}$

3. Use the method of elimination to solve the following simultaneous equations. 4.4

   (a) $\begin{cases} x + y = 8 \\ x - y = 2 \end{cases}$ **(b)** $\begin{cases} 4x + 5y = 22 \\ 3x + 2y = 13 \end{cases}$

4. **(a)** The sum of x and y is 95 and their difference is 51. If x is larger than y, set up simultaneous equations in x and y. 4.5

   (b) 4 kg of tea and 6 kg of coffee beans are sold for \$240.1 kg of tea and 8 kg of coffee beans are sold for \$242. Find the respective selling prices of tea and coffee beans per kg.

Revision Exercise 4

Level 1

1. Solve the following simultaneous equations graphically.

(a) $\begin{cases} y = x + 1 \\ y = 2x - 1 \end{cases}$ (b) $\begin{cases} 2x + y = 1 \\ x - 2y = 3 \end{cases}$ (c) $\begin{cases} x + 3y = 10 \\ 2x = y - 1 \end{cases}$

2. Use the method of substitution to solve the following simultaneous equations.

(a) $\begin{cases} x + y = 14 \\ 2x + 3y = 34 \end{cases}$ (b) $\begin{cases} 2x + y = 7 \\ 3x + 2y = 12 \end{cases}$ (c) $\begin{cases} x - 4y = 10 \\ 3x + 5y = 13 \end{cases}$

(d) $\begin{cases} 2x + 5y = -5 \\ 4x + 3y = -17 \end{cases}$ (e) $\begin{cases} 3x - 2y = 6 \\ 6x - 5y = 30 \end{cases}$ (f) $\begin{cases} 4x + 3y - 1 = 0 \\ 3x - 2y - 22 = 0 \end{cases}$

3. Use the method of elimination to solve the following simultaneous equations.

(a) $\begin{cases} 2x + y = 9 \\ 3x - y = 1 \end{cases}$ (b) $\begin{cases} x + 3y = 1 \\ 3x + y = 11 \end{cases}$ (c) $\begin{cases} x + 4y = 13 \\ 3x + 2y = 9 \end{cases}$

(d) $\begin{cases} 2x - 3y = 5 \\ 3x + 2y = 14 \end{cases}$ (e) $\begin{cases} 6x + 5y = 11 \\ 5x - 2y = 3 \end{cases}$ (f) $\begin{cases} 4x - 5y = 6 \\ 24x = 9y + 1 \end{cases}$

4. The sum of two numbers x and y is 23. The sum of 3 times x and 5 times y is 87. Find x and y.

5. The sum of two numbers x and y is 200. If $x + 20$ equals 4 times $y + 5$, find x and y.

6. The length of a rectangle is 2 times of its width. The perimeter is 42 cm. Find the area of the rectangle.

7. Henry pays \$1 176 for 30 packs of mini size cookies and 40 packs of regular size cookies. Monica pays \$768 for 40 packs of mini size cookies and 20 packs of regular size cookies. Find the selling price of each kind of cookies per pack.

Level 2

8. Solve the following simultaneous equations.

(a) $\begin{cases} 2x - 3y = 7 \\ 5y - 6x = -17 \end{cases}$ (b) $\begin{cases} 2.5x - 3.5y = 19 \\ 1.5x - 0.5y = 1 \end{cases}$ (c) $\begin{cases} 3x - 1 = 4(y + 1) \\ 7(2x + 3) = 6y \end{cases}$

(d) $\begin{cases} 5x - 8 = 6(y + 2) \\ 9(y - 5) = -6x - 21 \end{cases}$ (e) $\dfrac{x+9}{3} - \dfrac{y-10}{2} = x + \dfrac{5y}{2} = 4$ (f) $\dfrac{x}{3} + \dfrac{y}{4} - \dfrac{3}{2} = \dfrac{7x}{12} - \dfrac{y+12}{8} = 0$

9. Sandy plans to pay \$48 for two kinds of pens. If she buys 5 ballpoints and 7 highlighters, she will need \$2 more. If she buys 7 ballpoints and 5 highlighters, she will have \$2 left. Find the selling price of each kind of pen.

10. A Mathematics test consists of sections A and B. Each question in section A carries 2 marks. Each question in section B carries 3 marks. If Ken has answered 40 questions correctly and got a total of 92 marks, how many questions in each section has he answered correctly?

11. The sum of the hundreds digit and the tens digit of a three-digit number is 13. If the hundreds digit and the tens digit are interchanged, the new number is 90 less than the original number. It is given that the units digit of the original number is 4. Find the original number.

12. **(a)** **(i)** Solve the simultaneous equations $\begin{cases} 2x+3y=13 \\ 3x-y=3 \end{cases}$.

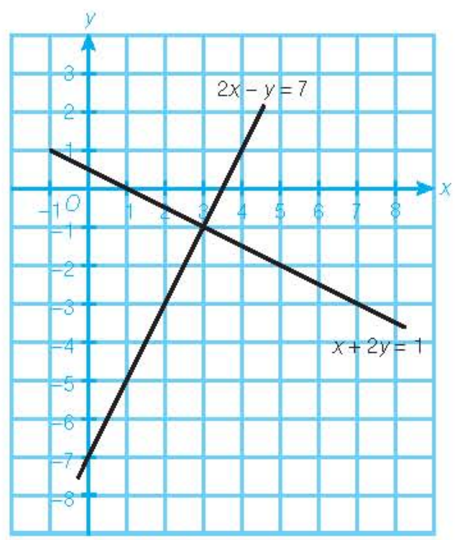
(ii) From the result of **(a)(i)**, solve the simultaneous equations $\begin{cases} 2(a+b)+3(a-b)=13 \\ 3(a+b)-(a-b)=3 \end{cases}$.

(b) **(i)** Solve the simultaneous equations $\begin{cases} 2x+y=10 \\ 4x+y=16 \end{cases}$.

(ii) From the result of **(b)(i)**, solve the simultaneous equations $\begin{cases} \dfrac{2}{a}+\dfrac{1}{b}=10 \\ \dfrac{4}{a}+\dfrac{1}{b}=16 \end{cases}$.

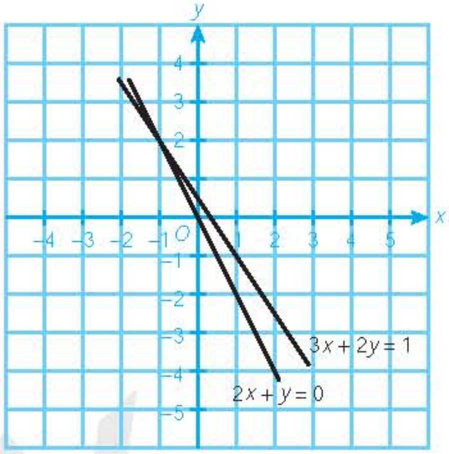
13. **(a)** According to the graph, solve the simultaneous equations $\begin{cases} x+2y=1 \\ 2x-y=7 \end{cases}$.

(b) Add a suitable straight line on the graph provided and solve the simultaneous equations $\begin{cases} x+2y=1 \\ x-y=7 \end{cases}$.



14. **(a)** According to the graph, solve the simultaneous equations $\begin{cases} 3x+2y=1 \\ 2x+y=0 \end{cases}$.

(b) Add a suitable straight line on the graph provided and solve the simultaneous equations $\begin{cases} 3x+2y=1 \\ 2x+3y=4 \end{cases}$.

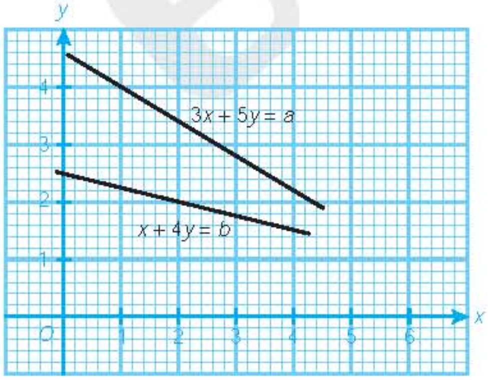


MC Question

15. The solution of the simultaneous equations $\begin{cases} x-7y=10 \\ x=2y \end{cases}$ is

A. $x=4$, $y=2$.
B. $x=2$, $y=1$.
C. $x=-2$, $y=-1$.
D. $x=-4$, $y=-2$.

16. According to the graph, find the solution of the simultaneous equations $\begin{cases} 3x+5y=a \\ x+4y=b \end{cases}$.



A. $x=4$, $y=2.2$
B. $x=5$, $y=1.6$
C. $x=6$, $y=1$
D. It cannot be found.

17. If the ordered pair $(-3, 3)$ satisfies the simultaneous equations $\begin{cases} ax+5y=3 \\ bx-4y=3 \end{cases}$, then

A. $a=-4$, $b=5$.
B. $a=-3$, $b=3$.
C. $a=4$, $b=-5$.
D. $a=9$, $b=6$.

18. The simultaneous equations $\begin{cases} x=y+1 \quad \ldots\ldots(1) \\ 4x+y=1 \quad \ldots\ldots(2) \end{cases}$ are given.

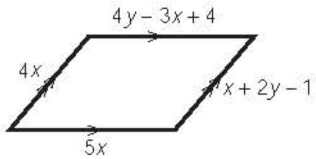
If we substitute equation (1) into equation (2), we have

A. $4x+x=1$.
B. $4x+y+1=1$.
C. $4(y+1)+y=1$.
D. $x=1-4x+1$.

19. If $3x - 5y - 14 = 5x - 7y - 38 = 0$, then $x - y =$

- **A.** 11.
- **B.** 12.
- **C.** 23.
- **D.** 34.

20. The figure shows a parallelogram. Find y.



- **A.** $\dfrac{11}{4}$
- **B.** 3
- **C.** 4
- **D.** 5

Problem-solving and Exploring

Hint for the Title Page Question

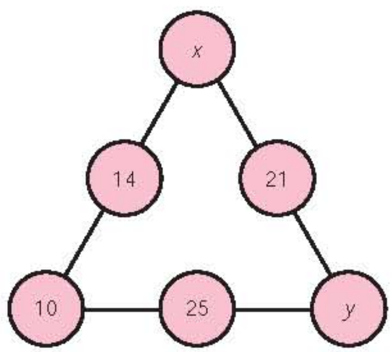
(a) If you did not send any SMS in a month, which plan would you choose to minimize the charge?

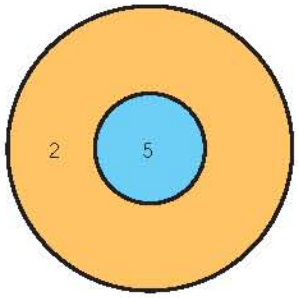
(b) If you sent 250 SMS in a month, which plan would you choose to minimize the charge?

(c) Suppose the charges for both plans were the same in a month, how many SMS did you send in that month?

Additional Question

1. In the figure, the sum of the numbers on each line are the same. Find x and y.





2. The rules of a darts match are as follows (the dartboard is shown on the right):

I. A target score is drawn.

II. The number of darts thrown by each participant is not limited.

III. A participant wins if the total score is equal to the target score.

(a) If 2 darts hit the region of the score of 5 and 3 darts hit the region of the score of 2, find the total score.

(b) Can Frankie obtain a total score of 23 by hitting the dartboard 7 times? Explain.

(c) Is it possible to obtain a total score of 30? Explain.

I.T. Corner

Graphical Solution of Simultaneous Linear Equations in Two Unknowns

Aim: To explore the graphical method for solving simultaneous linear equations in two unknowns by using graphical software

Required software: Graphmatica

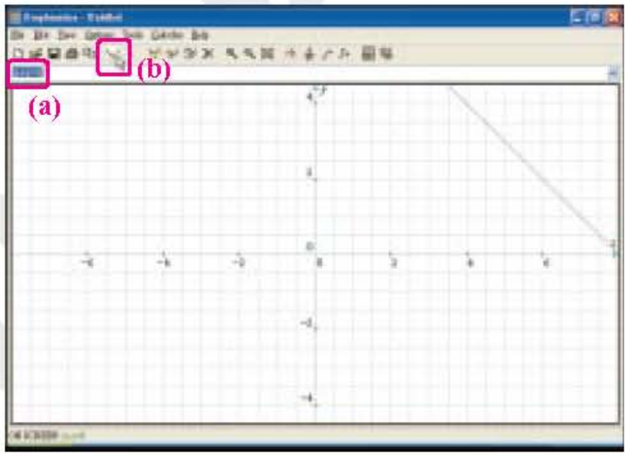
(*Note:* Download the trial version from the web site www.graphmatica.com)

Procedures:

Step 1: Type in the first equation

(a) Type in the first equation, e.g. '$x+y=8$', in the box under the tool bar.

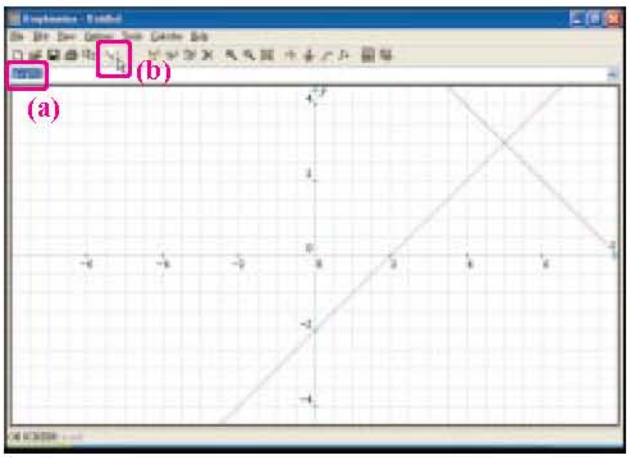
(b) Press [✓] or [Enter].



Step 2: Type in the second equation

(a) Type in the second equation, e.g. '$x-y=2$', in the box under the tool bar.

(b) Press [✓] or [Enter].

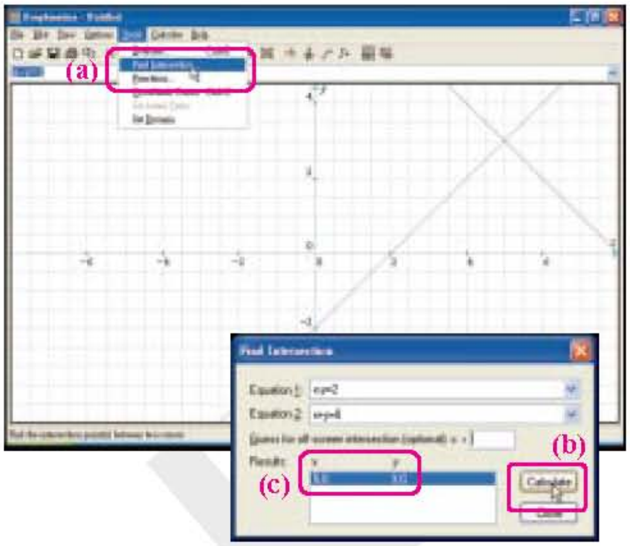


Step 3: Obtain the graphical solution

(a) Select [Tools] > [Find Intersection].

(b) Click [Calculate] button.

(c) The coordinates of the intersection of the graphs will be shown in the box under [Results].



Practice

Solve the following simultaneous linear equations in two unknowns by using Graphmatica.

(a) $\begin{cases} 4x + 5y = 6 \\ 7x + 2y = 9 \end{cases}$

(b) $\begin{cases} x + y = 5 \\ 5x - 2y = 6 \end{cases}$

Exploration with Fun — Enrichment

Special Simultaneous Linear Equations in Two Unknowns

Do all simultaneous linear equations in two unknowns have a unique solution? Are there any simultaneous equations that do not have a solution or with more than one solution?

A Inconsistent equations

Consider the following simultaneous equations.

$$\begin{cases} x+3y=6 \text{ .... } (1) \\ 2x+6y=6 \text{ .... } (2) \end{cases}$$

(1) × 2,

$2x+6y=12$ .... (3)

(3) − (2),

$$(2x+6y)-(2x+6y)=12-6$$
$$0=6$$

Clearly, '0 = 6' is not valid. What is the solution of the equations?

The figure below shows the graphs of the equations above.

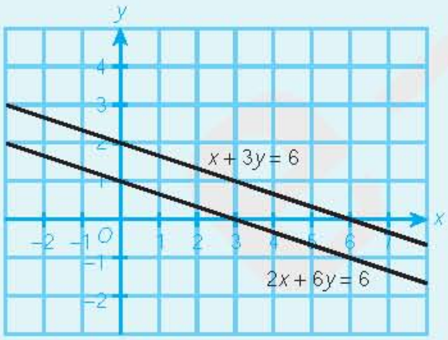


Figure 4.4

We can see that the graph above is a pair of parallel lines. Since they never intersect, there is no graphical solution.

As we can see from the discussion above, some simultaneous equations do not have a solution. We call these simultaneous equations **inconsistent equations**.

inconsistent equations 不相容方程

B Dependent equations

Consider the following simultaneous equations.

$$\begin{cases} 2x - y = 1 \text{ .... (1)} \\ 4x - 2y = 2 \text{ .... (2)} \end{cases}$$

(1) × 2,

$4x - 2y = 2$ .... (3)

(3) − (2),

$$(4x - 2y) - (4x - 2y) = 2 - 2$$
$$0 = 0$$

Clearly, '0 = 0' is always true. What is the solution of the equations?

The figure below shows the graphs of the equations above.

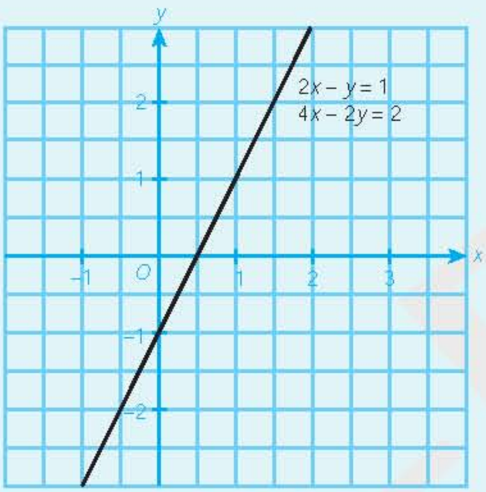


Figure 4.5

We can see that the graph above is a pair of overlapping lines, and so they have infinitely many points of intersection. In fact, the coordinates of all points on the line are solutions of the simultaneous equations.

As we can see from the discussion above, some simultaneous equations have infinitely many solutions. We call these simultaneous equations **dependent equations**.

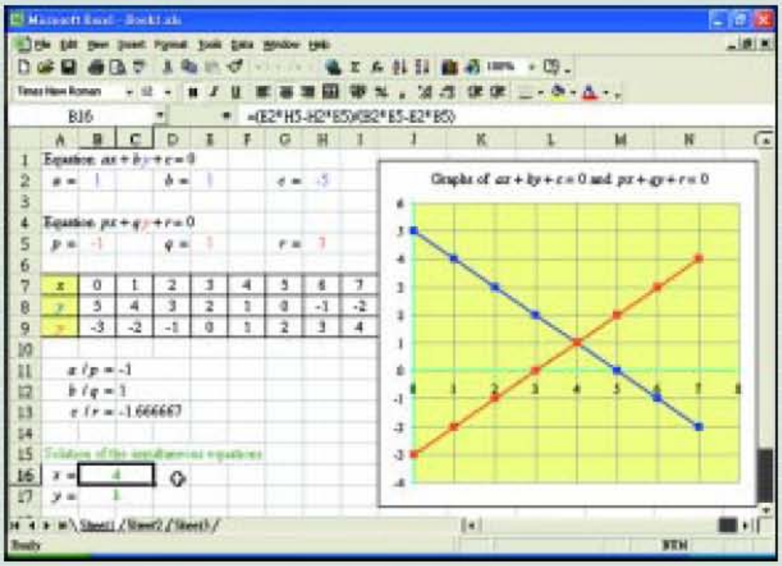
dependent equations 相關方程

In the following Class Activity, let us explore the relation between the coefficients of the simultaneous equations and the number of solutions they have.

Class Activity 4.3

Aim: To investigate the relation between the coefficients of the simultaneous equations and the number of solutions they have

The figure below shows the graph of the simultaneous equations $\begin{cases} ax+by+c=0 \\ px+qy+r=0 \end{cases}$ created with a spreadsheet, where $a=1$, $b=1$, $c=-5$, $p=-1$, $q=1$ and $r=3$.



The formula of cell B8 is '= -1*($B$2/$E$2)*B7-$H$2/$E$2'
The formula of cell B9 is '= -1*($B$5/$E$5)*B7-$H$5/$E$5'
The formula of cell C11 is '= B2/B5'
The formula of cell C12 is '= E2/E5'
The formula of cell C13 is '= H2/H5'
The formula of cell B16 is '= (E2*H5-H2*E5)/(B2*E5-E2*E5)'
The formula of cell B17 is '= (B2*H5-H2*B5)/(E2*B5-B2*E5)'

I.T. Tool

(a) In the figure, do the graph of the two equations only meet at one point?

☐ Yes ☐ No

Show Ans.

(b) Change the values of a, b, p and q such that $\frac{a}{p}=\frac{b}{q}$ and $\frac{b}{q}\neq\frac{c}{r}$. How does the graph change? How many points of intersection are there?

Show Ans.

____

____

(c) Change the values of a, b, c, p, q and r such that $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}$. How does the graph change? How many points of intersection are there?

Show Ans.

____

____

(d) Observe the coordinates of the points of intersection and the values of cells B16 and B17. What is special about them?

Show Ans.

____

____

Now I see ...

When $\frac{a}{p}=\frac{b}{q}=\frac{c}{r}$, the graphs of the two simultaneous equations coincide, and thus the simultaneous equations have infinitely many solutions. When $\frac{a}{p}=\frac{b}{q}\neq\frac{c}{r}$, the graphs of the simultaneous equations are parallel, and thus the simultaneous equations have no solution.

Extension 4.3

Solution

Determine which of the following are inconsistent equations, and which are dependent equations.

		Nature of equations
(a)	$\begin{cases} x+y=2 \\ 3x+3y=3 \end{cases}$	Inconsistent / Dependent
(b)	$\begin{cases} 2x-y=3 \\ 4x-2y=6 \end{cases}$	Inconsistent / Dependent
(c)	$\begin{cases} y=3x+1 \\ 3y=9x+3 \end{cases}$	Inconsistent / Dependent
(d)	$\begin{cases} x=4y-2 \\ 3x=12y-10 \end{cases}$	Inconsistent / Dependent
(e)	$\begin{cases} 7x-3y=5 \\ 14x-6y=5 \end{cases}$	Inconsistent / Dependent
(f)	$\begin{cases} 2y+3x=1 \\ y+\frac{3x}{2}=\frac{1}{2} \end{cases}$	Inconsistent / Dependent